

Messenger Plus! Forums question by Rubber Stamp

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The question:

$$\int (x + \frac{1}{x})(\sqrt{x^2 + \frac{1}{x^2}})dx \quad (1)$$

Let's add a square root to the first part of the formula:

$$\int (\sqrt{(x + \frac{1}{x})^2})(\sqrt{x^2 + \frac{1}{x^2}})dx \quad (2)$$

Removing the ² gives:

$$\int (\sqrt{(x + \frac{1}{x})(x + \frac{1}{x})})(\sqrt{x^2 + \frac{1}{x^2}})dx \quad (3)$$

Which is the same as:

$$\int (\sqrt{(x^2 + x * \frac{1}{x} + x * \frac{1}{x} + \frac{1}{x} * \frac{1}{x})})(\sqrt{x^2 + \frac{1}{x^2}})dx \quad (4)$$

And when we simplify that, it is:

$$\int (\sqrt{(x^2 + \frac{1}{x^2} + 2)})(\sqrt{x^2 + \frac{1}{x^2}})dx \quad (5)$$

There's a mathematical rule stating that:

$$\sqrt{a} * \sqrt{b} = \sqrt{a * b} \quad (6)$$

For example $\sqrt{4} * \sqrt{9} = \sqrt{36}$. Using that, we can change formula 5 to:

$$\int \sqrt{(x^2 + \frac{1}{x^2} + 2)(x^2 + \frac{1}{x^2})}dx \quad (7)$$

And the obvious following step is to do:

$$\int \sqrt{(x^2 * x^2 + x^2 * \frac{1}{x^2} + \frac{1}{x^2} * x^2 + \frac{1}{x^2} * \frac{1}{x^2} + 2 * x^2 + 2 * \frac{1}{x^2})}dx \quad (8)$$

Simplifying that gives:

$$\int \sqrt{(x^4 + 2x^2 + 2 + \frac{2}{x^2} + \frac{1}{x^4})}dx \quad (9)$$